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PRECISE MEASUREMENTS OF THE WAVELENGTH AT THE ONSET OF RAYLEIGH–BENARD CONVECTION IN A LONG RECTANGULAR DUCT

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NOMENCLATURE

d,	depth of the fluid layer;
g,	acceleration due to gravity;
l,	width of the fluid layer;
L,	length of the fluid layer;
N,	number of convective rolls;
Ra,	Rayleigh number, $(g\ \alpha_T\ \Delta T\ d^3/\kappa\nu)$;
Ra^{cr},	critical Rayleigh number;
V_x, V_y, V_z,	cartesian components of velocity;
x, y, z,	cartesian coordinates.

Greek symbols

α,	wavenumber in the x-direction;
α_T,	coefficient of volume expansion;
β,	width-to-height ratio of the apparatus, l/d;
γ,	length-to-width ratio of the apparatus L/l;
ΔT,	temperature difference between the two horizontal plates;
ν,	kinematic viscosity of the fluid;
κ,	thermal diffusivity of the fluid.

1. INTRODUCTION

WHEN a fluid layer is heated uniformly from below, the state of rest becomes unstable when the critical Rayleigh number Ra^{cr} (the non-dimensional temperature gradient) exceeds a certain value. At the onset of convection, the shape of the convective motion depends on the nature and the geometry of the boundaries.

In the case of a rectangular channel, various theoretical works have been achieved; they all show that convection appears under the form of rolls aligned with their axis perpendicular to the axis of the channel [1–5].

The results concerning the onset of free convection in an infinite rectangular duct with four rigid boundaries, two perfectly conducting horizontal plates and two insulating lateral walls have been given elsewhere [5]. There the stability of the following cases was studied:

(i) infinite longitudinal rolls [with their axis parallel to the axis of the channel ($V_x \equiv 0$ in Fig. 1)].

(ii) finite transverse rolls ($V_y \equiv 0$),

(iii) 3-dim. rolls (V_x, V_y, $V_z \neq 0$).

Three-dimensional perturbations lead to the lowest critical Rayleigh numbers. At the onset of convection, the existence of a y-component of the velocity leads to a z-symmetrical y-curvature of the trajectories of fluid particles. The dependence of the critical wavenumber (related to the size of convective rolls at the critical point), on the width-to-height ratio is very characteristic of the 3-dim. structure: in the case of longitudinal rolls α^{cr} is obviously equal to zero; for finite transverse rolls, it decreases monotonically when the width-to-height ratio increases (curve a, Fig. 2) and for three-dimensional rolls one obtains the particular curve b on Fig. 2. This characteristic shape does not depend on the nature of the horizontal walls since it has been sketched as well as for 'free–free' boundary conditions [3], and for 'rigid–rigid' ones [5].

Concerning the experimental point of view, Oertel and Bühler [6] quoted, via a differential interferometry technique, the presence of a three-dimensional convective motion in a duct with a length-to-width ratio (γ) of 2.5 and Rayleigh numbers not less than 1.5 times the critical Rayleigh number. In this work we intend to measure the wavelength of convective cells as a function of the width-to-height ratio in a very long apparatus ($\gamma \geq 17.5$) and as close as possible to the threshold of Rayleigh–Bénard convection. For that, we use the same experimental apparatus and the same shadowgraph method described earlier [7] in a paper about the influence of a superimposed basic flow on the shape of thermoconvective rolls in a long duct.

2. EXPERIMENTAL DETAILS

The apparatus that we used in a previous work [7] will be described shortly again for convenience (Fig. 3). A rectangular Plexiglas frame is enclosed between two 3 cm thick copper plates kept at constant temperature by flows of thermoregulated water. The flow rate is about $50\,cm^3\,s^{-1}$. Available space for the fluid is: length $L = 93.5$ cm between the two porous media, height $d = 1$ cm and the width can be varied from 0 to 5.25 cm. As usual in this kind of experiment, we used a silicone oil, a very good Boussinesq fluid. In particular, we employed the Dow Corning 200 fluid with a viscosity equal to $0.50 \pm 0.01\,cm^2\,s^{-1}$. The Prandtl number is about 500. Convection is visualized by a very simple shadow graph method, that is the so-called thermal lens effect. The image of the convective rolls on a screen located at the focal length (whose position depends on the velocity of the convective motion) presents thus a succession of alternately dark and pale areas, the number of which representing the number of convective rolls in the apparatus. A typical photograph of the image of the convective rolls was shown in our previous paper [7].

3. RESULTS

First of all, we tried to determine the critical Rayleigh number (Ra^{cr}) and the corresponding critical wavenumber

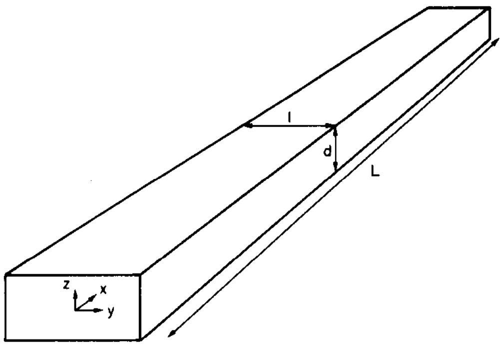

FIG. 1. Coordinate system.

temperature difference was found to be 0.925 ± 0.05°C. Knowing the physical properties of the fluid at 25°C (α_T = 1.04 ± 0.01 10^{-3} K^{-1}, κ = 1.10 ± 0.03 10^{-3} $cm^2 s^{-1}$, ν = 0.50 ± 0.01 $cm^2 s^{-1}$) allows the calculation of the corresponding Rayleigh number (d = 1.00 ± 0.01 cm)

$$Ra^{cr} = \frac{g\,\alpha_T \Delta T^{cr} d^3}{\kappa\,\nu}$$

$$\cong \frac{981 \times 1.04 \times 0.925 \times 1^3}{1.10 \times 0.50} = 1716$$

to compare with theoretical value of 1711.49 [5]. This rather good agreement is fortuitous, the experimental error on Ra^{cr} being ± 200.

We also counted the presence of 92 convective rolls for this aspect ratio at the onset of convection (this experiment was repeated five times within several weeks each time for 92 rolls). The corresponding wavenumber is thus

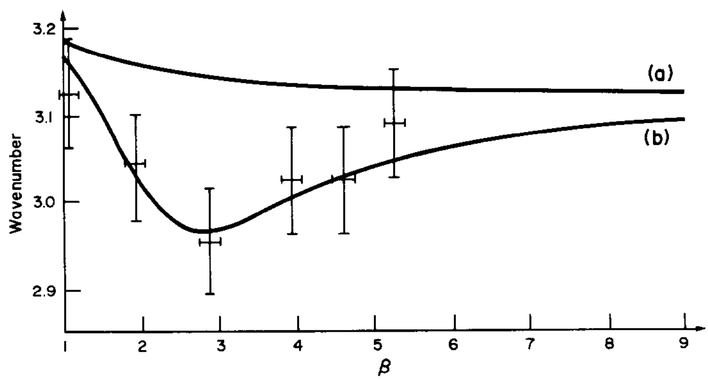

FIG. 2. Critical wavenumber vs the width-to-height ratio of the apparatus. (a) Theory: finite rolls assumption. (b) Theory: 3-dim. perturbations. Experimental values with their error are also represented.

(α^{cr}) for a particular width-to-height ratio, namely β = 5.25. Theoretical values are: Ra^{cr} = 1711.49 for β = 5.25 [5] instead of Ra^{cr} = 1707.762 for $\beta \to \infty$ [8].

The procedure used was as follows: the lower plate was thermoregulated at 25 ± 0.05 °C and the upper boundary at 24.50 ± 0.05°C. The temperature of the upper plate was decreased by steps of about 0.05°C. Roll images appear on the screen after a few hours. (Preliminary crude experiments were performed to determine when threshold of convection occurs). Convection always starts at the boundaries (near the porous media), about twenty minutes after the critical gradient has been imposed. The convective motion was then allowed to fill the fluid layer completely (typically a few hours). At this point the number of convective rolls and the temperature difference between the two baths of thermoregulated water was recorded. For β = 5.25, this critical

$$\alpha^{cr} = (\pi/L)\,N\,d = \frac{\pi}{93.5\,\text{cm}} \times 92 \times 1\,\text{cm} = 3.09$$

the theoretical value being α^{cr} = 3.043 for 3-dim. perturbations.

Experimental error on α^{cr} may be a result of the following causes:

(i) Non-parallelism of the boundaries and error on the measurement of d (of less than 1%).

(ii) Relative error of $1/N$ due to the fact that the channel is not infinitely long. (Obviously the number of rolls in the experiment must be an integer).

(iii) The 'critical point' was judged to have been reached when an image of a well structured 'macroscopic' rolls system was seen on the screen within a few hours. Of course this 'critical temperature gradient' exceeds that of the linear

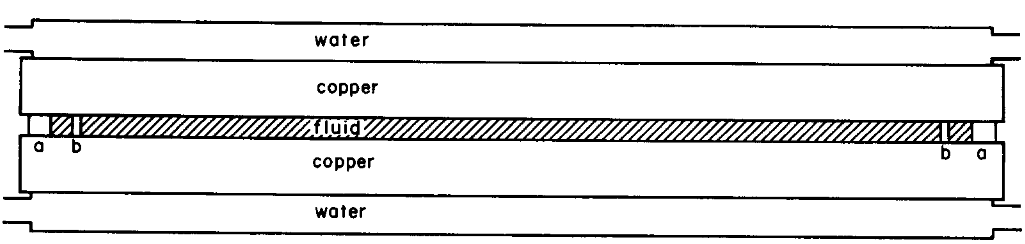

FIG. 3. Experimental set-up. (a) Plexiglas frame. (b) Porous media.

Table 1.

$\beta = l/d$	Number of rolls			Wavenumber		
		Theory*			Theory	
	Experiment	(a)	(b)	Experiment	(a)	(b)
1.07	93	94	95	3.125	3.162	3.185
1.94	90 or 91	90	94	3.041	3.027	3.160
2.88	88	88	94	2.957	2.966	3.145
3.93	90	89	93	3.024	3.001	3.136
4.59	90	90	93	3.024	3.024	3.132
5.25	92	91	93	3.091	3.043	3.129

*The number of rolls calculated has been rounded off.
(a) Three-dimensional perturbations.
(b) Finite rolls approximation.

theory (where the most unstable infinitesimal disturbances neither grow nor decay). Probably what we call 'critical point' is characterised by a temperature gradient $\Delta T \simeq 1.05\ \Delta T^{cr}$. This may induce a systematic error. In fact, the wavenumber was found to decrease when the temperature gradient increases. For this aspect ratio, 2 rolls were lost for $\Delta T \simeq 1.3\ \Delta T^{cr}$, 3 rolls for $\Delta T \simeq 1.8\ \Delta T^{cr}$ and 18 rolls for $\Delta T \simeq 9.2\ \Delta T^{cr}$). However, it is likely that, at $\Delta T \simeq 1.05\ \Delta T^{cr}$, the number of convective rolls is the same at the 'true' critical temperature gradient.

Taking into account the two first sources of errors the error on the critical wavenumber is less than 2%.

So

$$\alpha^{cr}\ (\beta = 5.25) = 3.09 \pm 0.05.$$

We undertook the same determination of α^{cr} for six width-to-height ratios. Table 1 gives the observed number of rolls and the resulting wavenumber compared with the theoretical values, based on our paper [5]. In Fig. 2, curve (a) shows the critical wavenumbers for transverse finite rolls ($V_y \equiv 0$) and curve (b) is relative to the onset of '3-dim. convection'. These experiments have been performed two or three times for each aspect ratio and always conducted to the same number of rolls at the critical point, except for the case $\beta = 1.94$, where two successive runs 90 and 91 rolls were observed, giving two experimental values for this aspect ratio in Table 1.

4. CONCLUSION

We conclude from Fig. 2 that, when the width-to-height ratio of a long fluid layer increases, the corresponding critical wavenumber begins to decrease, reaches a minimum for $\beta \simeq 3$ and then increases. It also appears that these experimental data fit closely the curve corresponding to 3-dim. perturbations and disagree with the finite rolls approximation, providing a supplementary, but indirect proof of the 3-dim. character of the flow: there should exist a non-zero y-velocity component in the Rayleigh–Bénard convective motion even near the threshold of convection.

We have also proved that, contrary to the recent results of Frick and Clever [4], the difference between the initial values of the finite rolls assumption and the critical values of the 3-dim. theory is not negligible and can be experimentally measured.

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